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RAN-2101001102010001

M. A. (Mathematics) Part-II (External) Examination March - 2026

Differential Geometry and Linear Algebra

[Total Marks: 100

સૂચના : / Instructions	Seat No.:
(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) All questions are compulsory. (3) Follow usual notations and conventions (4) Figures to the right indicate marks of the question.	Student's Signature

Q. 1. (a) Define Inflexional tangent. Find the equation of plane that has three point contact at the origin withthe curve $x = u^4 - 1$, $y = u^3 - 1$ and $z = u^2 - 1$. **7**

(b) Show that a necessary and sufficient condition for a curve to be a straight line is that $K = 0$ at all points. **7**

(c) Calculate the curvature and torsion of the curve given by $r = (u, u^2, u^3)$. **6**

OR

Q. 1. (a) State and prove Serret-Frenet formulae. **7**

(b) Show that a necessary and sufficient condition for a curve to be a helix is that the ratio of the curvature and the torsion is constant. **7**

(c) Show that the curvature of the helix $x = a\cos\theta$, $y = a\sin\theta$, $z = a\theta\tan\alpha$ is $\dfrac{\cos^2\alpha}{a}$. **6**

Q. 2. (a) Define Involute. Find the curvature and torsion of the involute of the curve. **7**

(b) For Bertnard curves C and C_1, if P and Q are corresponding points of C and C_1, prove that (i) the distance PQ is constant, (ii) the tangent at P to C makes a constant angle with the tangent at Q to C_1. **7**

(c) Find the equation of the osculating sphere and osculating circle at (1,2,3) on the curve $x = 2t + 1$, $y = 3t^2 + 2$, $z = 4t^3 + 3$. **6**

OR

Q. 2. (a) Show that the sum of squares of the intercepts on the coordinate axes made by the tangent plane to the surface $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$ is constant. **7**

(b) Prove that $F = 0$ is the necessary and sufficient condition for the paramentric curve on the surface to be orthogonal. **7**

(c) Find an equation for the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$. **6**

Q. 3. (a) Prove that the envelope of a family of surfaces touches each member of the family, at all points of its characteristic. **7**

(b) Calculate the fundamental magnitudes for the coincoid $x = u\cos v$, $y = u\sin v$, $z = f(v)$, with u, v as parameters. **7**

(c) By considering the value of $rt - s^2$, determine if the surface $xyz = a^3$ is developable. **6**

OR

Q. 3. (a) Let $T : V \rightarrow W$ be a linear operator. Prove that: **7**

(i) If T has an inverse S, then S is unique,

(ii) If T has left inverse, then T is one to one.

(iii) If T has right inverse $R : W \rightarrow V$, then T is onto,

(iv) T has an inverse if and only if T is one to one and onto.

(b) State and prove Rank-Nullity theorem. **7**

(c) Let T be amatrix operator $T(x) = Ax$, with $A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$. Find a basis for $N(T)$. Also find $r(T)$. **6**

Q. 4. (a) Let $T : V \rightarrow W$ and $S : W \rightarrow U$ be linear operators, where V, W, U are vector spaces and V is finite dimensional. Prove that $r(ST) \leq \min\{r(S), r(T)\}$. **7**

(b) State and prove QR factorization theorem. **7**

(c) Let $M = \text{span}\{v_1, v_2, v_3\}$, where $v_1 = (1, 1, 0, 1)^T$, $v_2 = (3,1, 2, -1)^T$ and $v_3 = (1, 2, 2, 0)^T$. Find the projection of f on M; where $f = (0, 1, -1, -1)^T$ using standard inner product. **6**

OR

Q. 4. (a) Define Inner product and Norm. Prove that $\langle x, y \rangle = \frac{1}{4}\{\|x + y\|^2 - \|x - y\|^2\}$, for all x, y in real vector space V. **7**

(b) For the subspace $M = \{x : x_1 + x_2 + x_3 + x_4 = 0, 2x_1 + x_2 = 0\}$ of R^4, find an orthogonal basis. **7**

(c) Find the QR decomposition for the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}$. **6**

Q. 5. (a) Let G be the Gram matrix of $\{v_1, v_2, \ldots, v_n\}$. Prove that the rank of G is the dimension of span $\{v_1, v_2, \ldots, v_n\}$. **7**

(b) Let M be the plane spanned by $v_1 = (1, -1, 0)^T$ and $v_2 = (0, 1, -1)^T$ in R^3. Find the projection operator on M and its matrix in the standard basis. **7**

(c) Find the vector in $M = \text{span}\{E_1, E_2, E_3\}$ nearest $f = (2, -2, -2, 6)^T$ where $E_1 = (1, 1, 1, 1)^T$, $E_2 = (1, -1, 0, 0)^T$ and $E_3 = (1,1, 0, -2)^T$. Also find the distance from f to M. **6**

OR

Q. 5. (a) Let A be any $m \times n$ matrix and let $y \in F^m$. Prove that exactly one of the following is true: **7**

(i) $Ax = y$ is consistent.

(ii) There exists z such that $A^H z = 0$ and $z^H y \neq 0$.

(b) Let V be a finite dimensional vector space and let M be a finite dimensional subspace of V. Prove that (i) $V = M \oplus M^{\perp}$ (ii) $(M^{\perp})^{\perp} = M$. **7**

(c) Let $A_1 = (1,2,3)^T$, $A_2 = (1, 0, 1)^T$, $A_3 = (-1, 2, 1)^T$. Find the Gram matrix of $\{A_1, A_2, A_3\}$ in the standard inner product and its rank. 6